

Training Large Language Models to Reason in a Continuous Latent Space

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Reason in latent space

- Traditional Chain-of-Thought (CoT) performs reasoning in language space (discrete tokens)



Solve the quadratic equation $x^2 - 5x + 6 = 0$



As a text-based AI, I am unable to directly solve quadratic equations. However, I can guide you on how to solve it using various methods. One common method is factoring. Let's try factoring the given quadratic equation $x^2 - 5x + 6 = 0$:

1. Factor the left side of the equation: $(x - 2)(x - 3)$
2. Set each factor equal to zero and solve for x :
 - $x - 2 = 0 \Rightarrow x = 2$
 - $x - 3 = 0 \Rightarrow x = 3$

So, the solutions to the quadratic equation $x^2 - 5x + 6 = 0$ are $x = 2$ and $x = 3$.

Why is language space potentially suboptimal?



Clinical: People can perform reasoning even when language is lost (aphasia)



Neuroimaging: The brain's language network stays quiet during non-linguistic reasoning (fMRI)

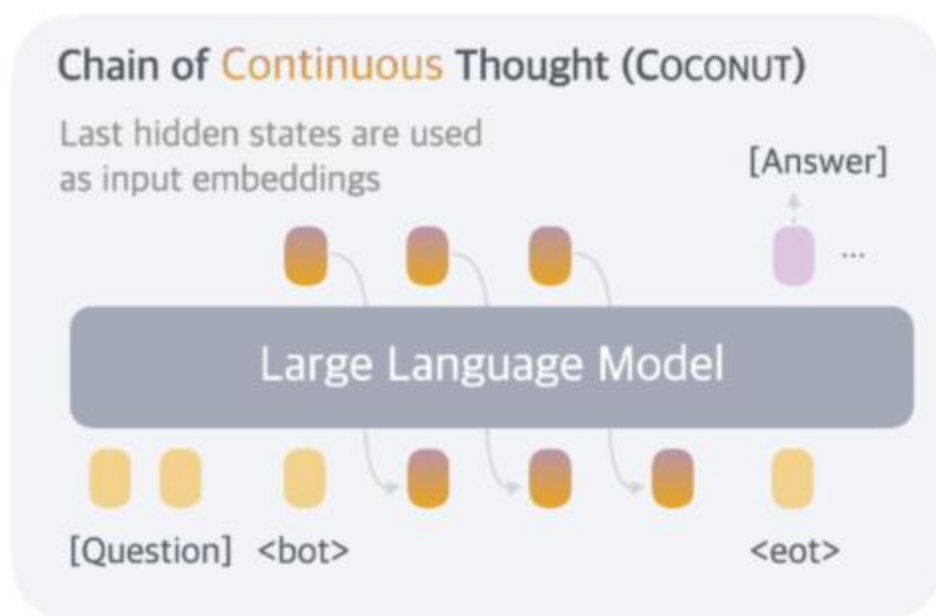
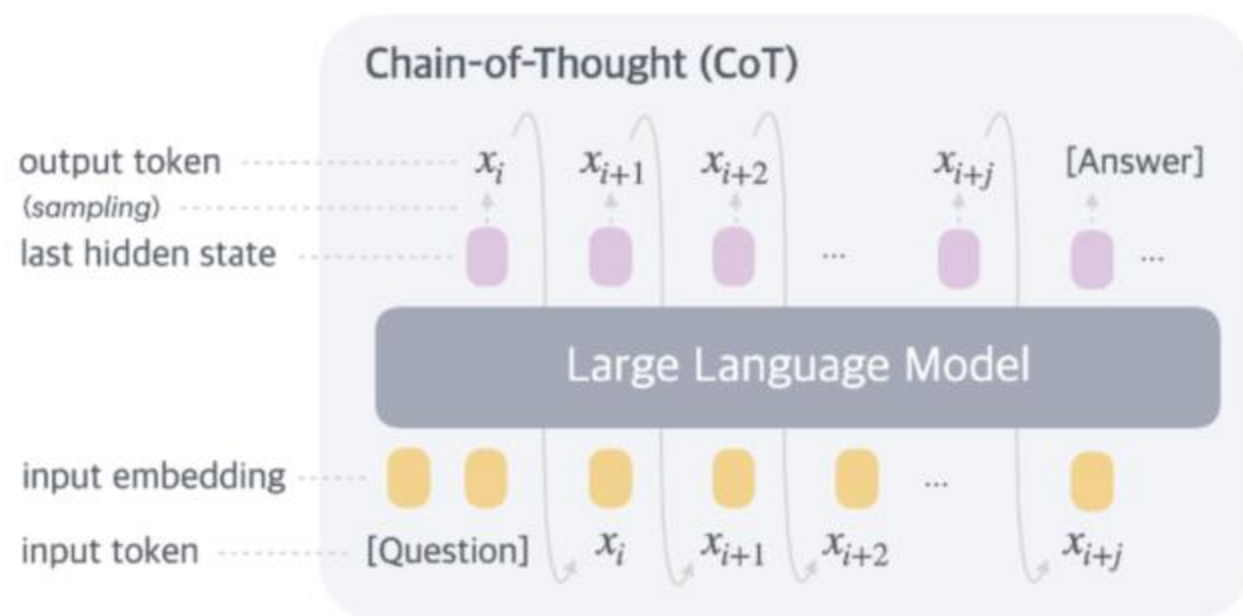


Linguistic: Syntactic and word constraints in language are optimized for communication efficiency, not for reasoning.

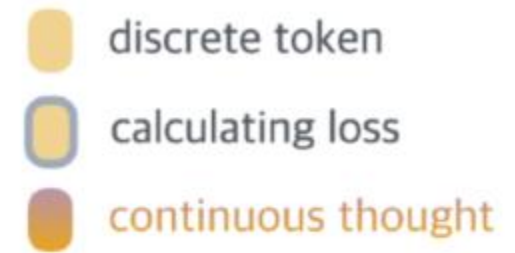
Why is language space potentially suboptimal (for LLMs)?

- Many tokens are only for fluency, not for “real” reasoning.
- Committing to one word at a time enforces a single reasoning path.

From Language to Latent Space



Training



CoT
(Supervised
fine-tuning)



A potential training objective...

CoCONUT



Experiments show this method cannot effectively guide the model to switch to latent space reasoning

Multi-stage training curriculum

CoT

(Supervised fine-tuning)



COCONUT

(Stage 0)



(Stage 1)



...

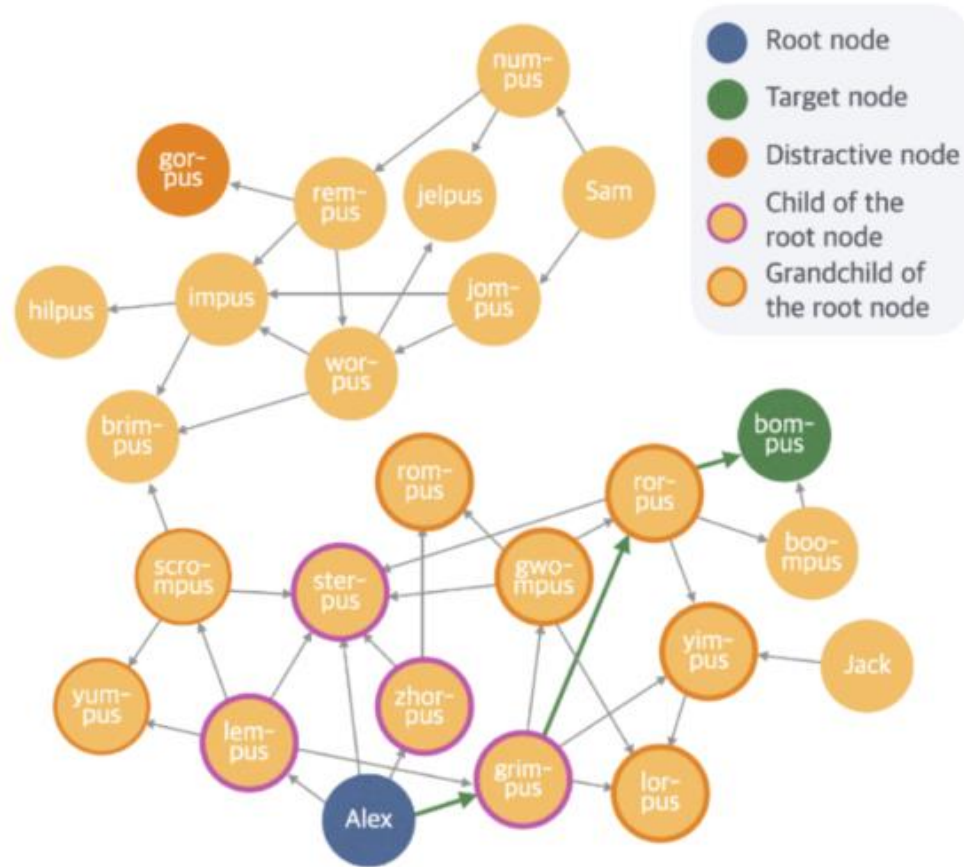
(Stage N)



$N \times$ continuous thoughts

- discrete token
- calculating loss
- continuous thought

Continuous Space Enables Latent Tree Search

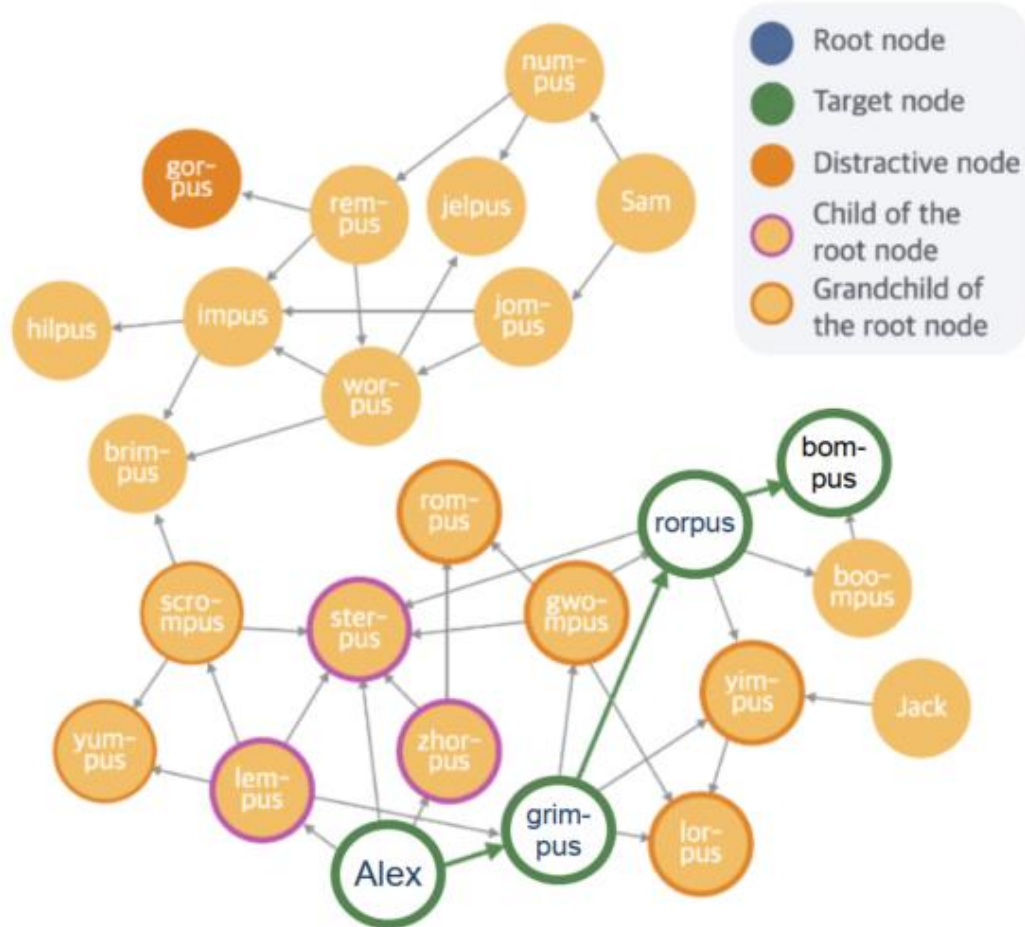


Question:

Every grimpus is a yimpus. Every worpus is a jelpus. Every zhorpus is a sterpus. Alex is a grimpus ... Every lumps is a yumpus.

Question: Is Alex a gorpus or bompus?

Continuous Space Enables Latent Tree Search



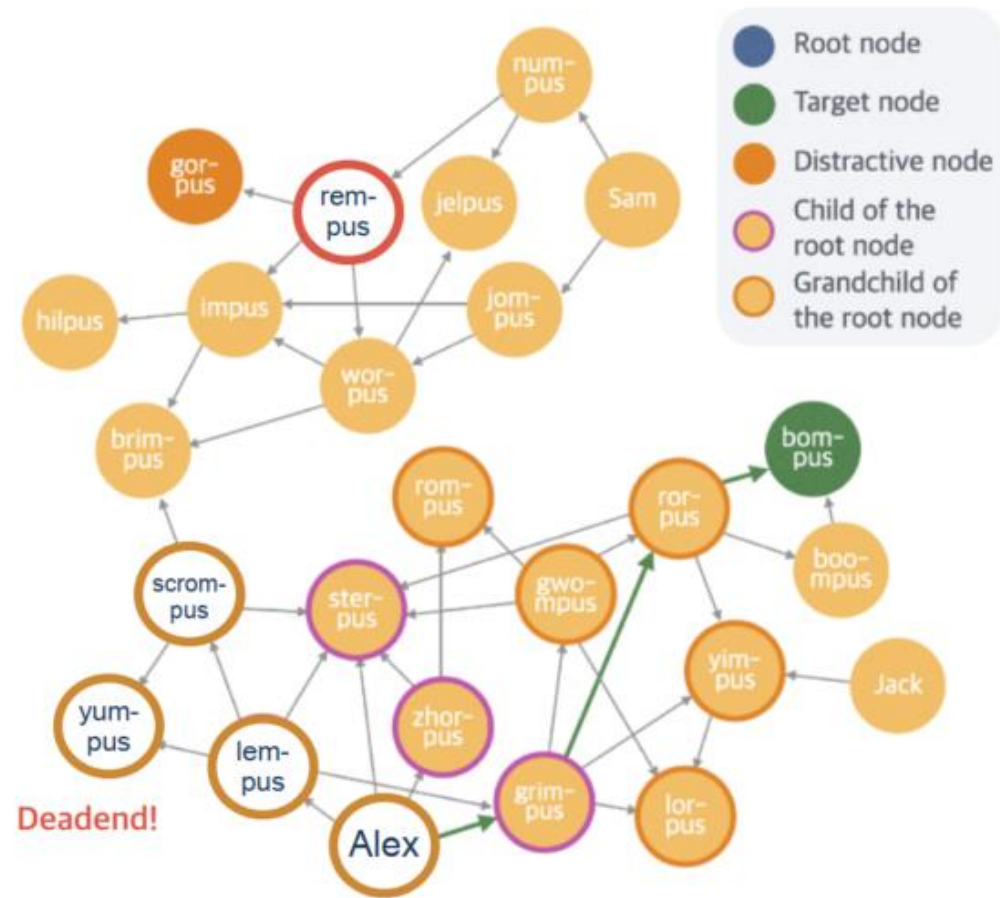
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Question: **Is Alex a gorpus or bompus?**

Ground Truth Solution

Alex is a grimpus.
Every grimpus is a rorpus.
Every rorpus is a bompus.
Alex is a bompus

Continuous Space Enables Latent Tree Search



Question:

Every grimpus is a yimpus. Every worpus is a jelpus. Every zhorpus is a sterpus. Alex is a grimpus ... Every lumpus is a yumpus.

Question: Is Alex a gorpus or bompus?

Ground Truth Solution

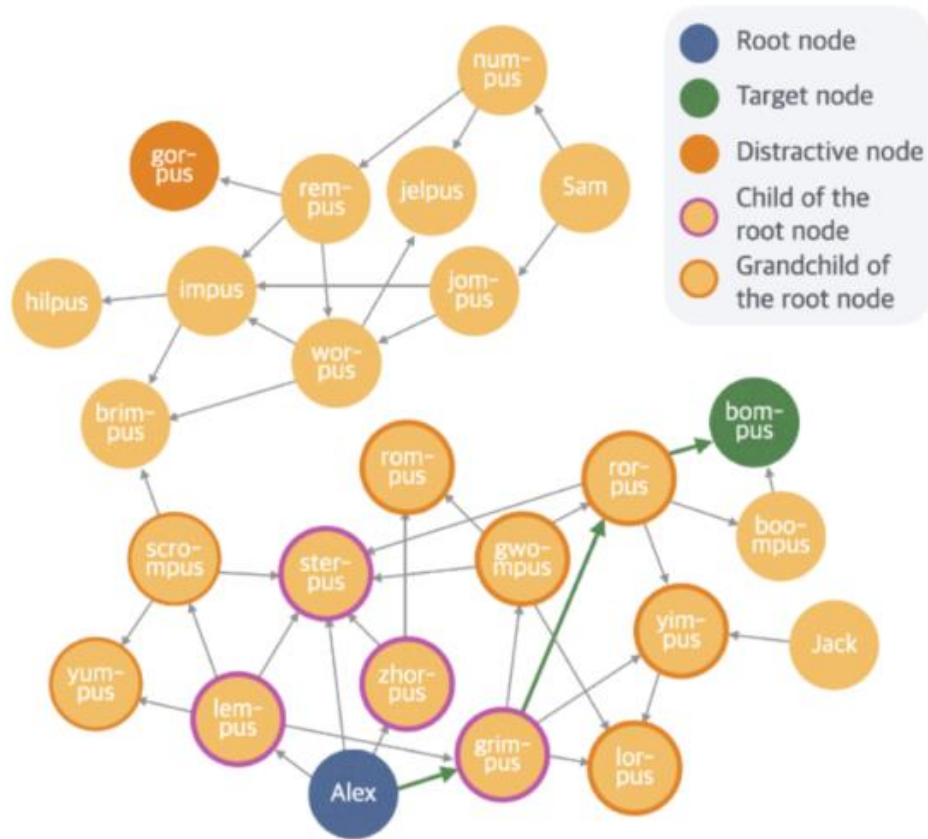
Alex is a grimpus.
Every grimpus is a rorpus.
Every rorpus is a bompus.
Alex is a bompus

CoT

Alex is a lempus.
Every lempus is a scrompus.
Every scrompus is a yumpus.
Every yumpus is a rempus.
Every rempus is a gorpus.
Alex is a gorpus ❌

(Hallucination)

Continuous Space Enables Latent Tree Search



Question:

Every grimpus is a yimpus. Every worpus is a jelpus. Every zhorpus is a sterpus. Alex is a grimpus ... Every lumpus is a yumpus.
Question: Is Alex a gorpus or bompus?

Ground Truth Solution

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CoT

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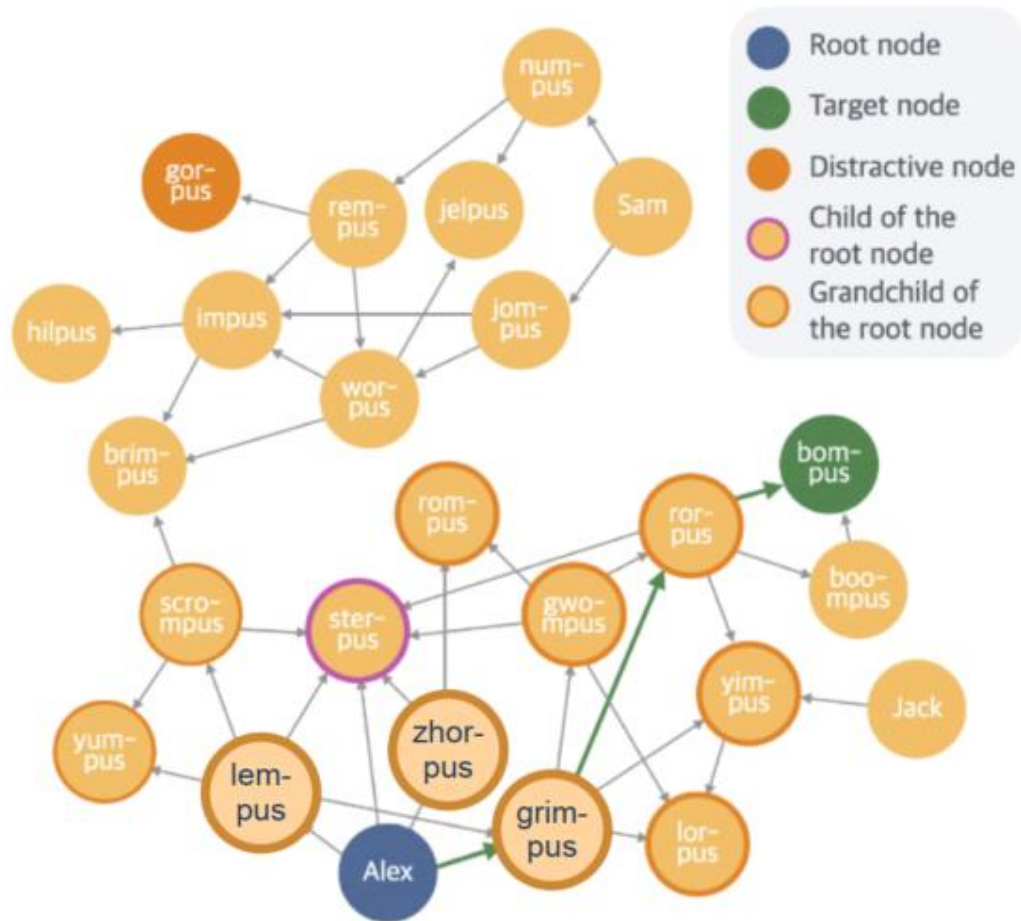
(Hallucination)

COCONUT (k=1)

<bot> [Thought] <eot>
Every lempus is a scrompus.
Every scrompus is a brimpus.
Alex is a brimpus ❌

(Wrong Target)

Continuous Space Enables Latent Tree Search



Question:

Every grimpus is a yimpus. Every worpus is a jelpus. Every zhorpus is a sterpus. Alex is a grimpus ... Every lumpus is a yumpus.
Question: Is Alex a gorpus or bompus?

Ground Truth Solution

Alex is a grimpus.
Every grimpus is a rorpus.
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Alex is a bompus

CoT

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Alex is a gorpus ❌

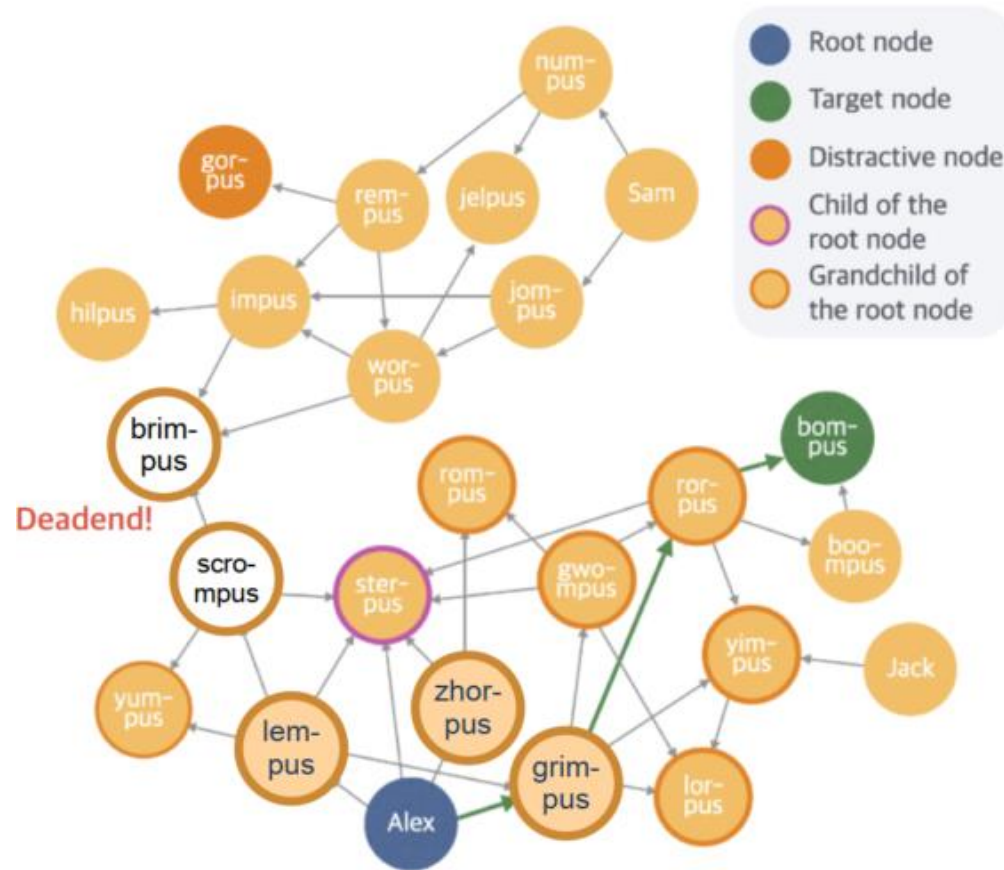
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(Wrong Target)

Continuous Space Enables Latent Tree Search



Question:

Every grimpus is a yimpus. Every worpus is a jelpus. Every zhorpus is a sterpus. Alex is a grimpus ... Every lumpus is a yumpus.
Question: Is Alex a gorp~~us~~ or bom~~pus~~?

Ground Truth Solution

Alex is a grimpus.
Every grimpus is a rorpus.
Every rorpus is a bompus.
Alex is a bompus

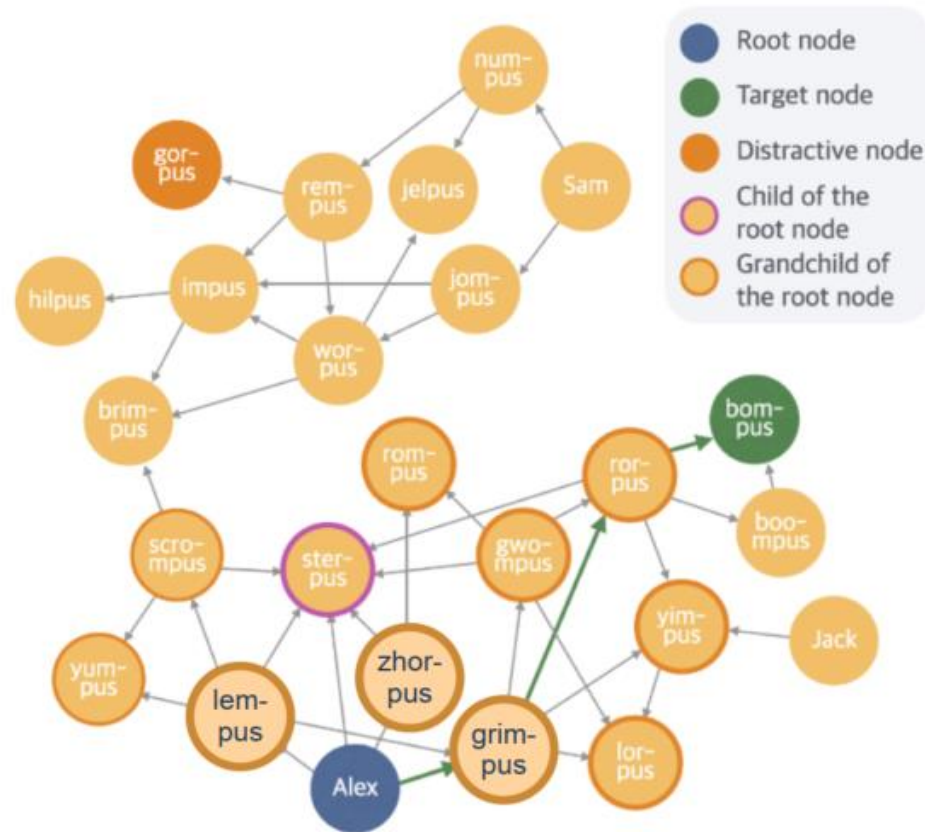
CoT

Alex is a lempus.
Every lempus is a scrompus.
Every scrompus is a yumpus.
Every yumpus is a rempus.
Every rempus is a gorp~~us~~.
Alex is a gorp~~us~~ ✗
(Hallucination)

COCONUT (k=1)

<bot> [Thought] <eot>
Every lempus is a scrompus.
Every scrompus is a brimpus.
Alex is a brimpus ✗
(Wrong Target)

Continuous Space Enables Latent Tree Search



Question:

Every grimpus is a yimpus. Every worpus is a jelpus. Every zhorpus is a sterpus. Alex is a grimpus ... Every lumps is a yumpus.
Question: Is Alex a gorpup or bompup?

Ground Truth Solution

Alex is a grimpus.
Every grimpus is a rorpus.
Every rorpus is a bompup.
Alex is a bompup

Coconut (k=1)

<bot> [Thought] <eot>
Every lempus is a scrompus.
Every scrompus is a brimpus.
Alex is a brimpus ❌

(Wrong Target)

CoT

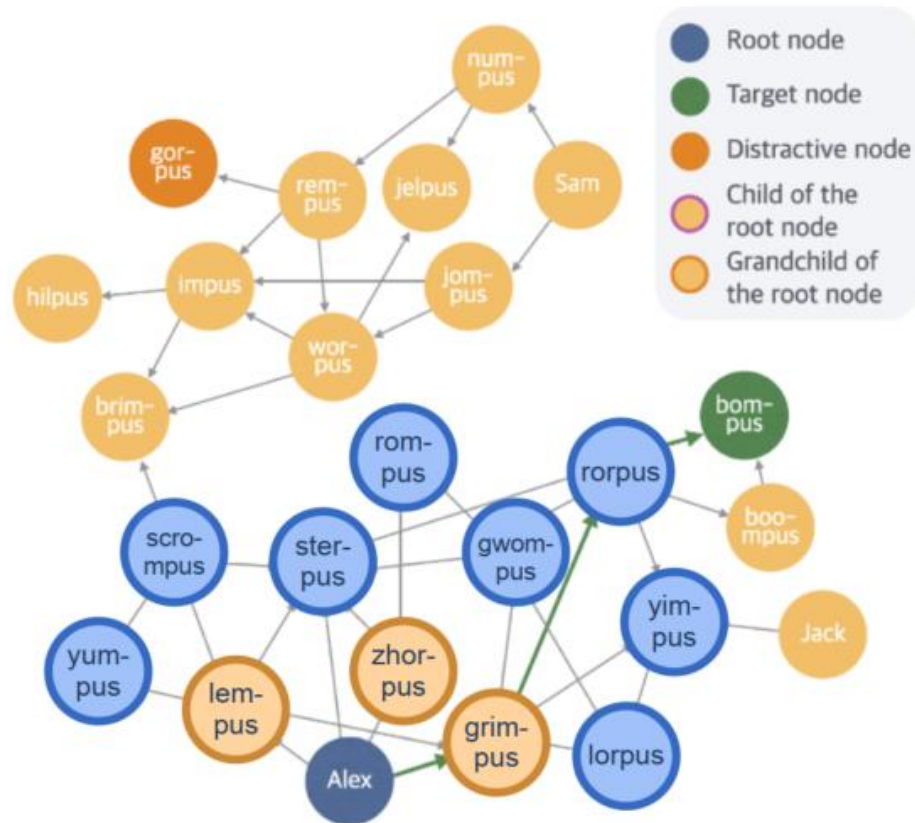
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Every yumpus is a rempus.
Every rempus is a gorpup.
Alex is a gorpup ❌
(Hallucination)

Coconut (k=2)

<bot> [Thought] [Thought] <eot>
Every rorpus is a bompup.
Alex is a bompup ✅

(Correct Path)

Continuous Space Enables Latent Tree Search



Question:

Every grimpus is a yimpus. Every worpus is a jelpus. Every zhorpus is a sterpus. Alex is a grimpus ... Every lumps is a yumpus.
Question: Is Alex a gorpus or bompus?

Ground Truth Solution

Alex is a grimpus.
Every grimpus is a rorpus.
Every rorpus is a bompus.
Alex is a bompus

CoCONUT (k=1)

<bot> [Thought] <eot>
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Alex is a brimpus ❌

(Wrong Target)

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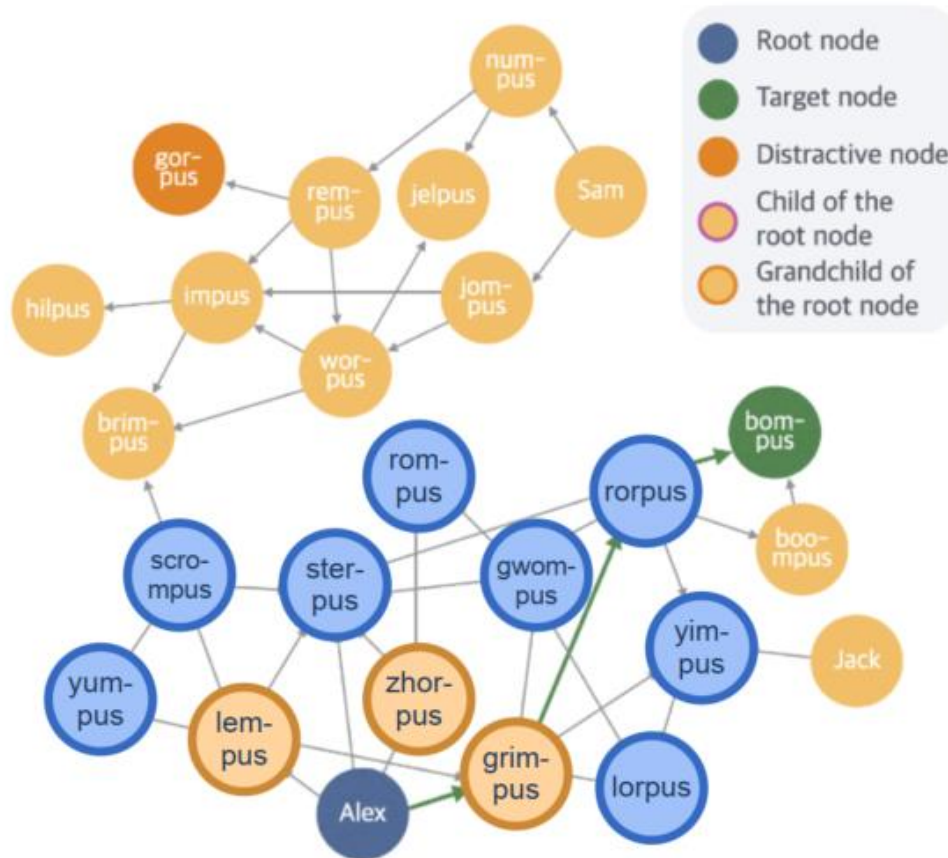
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CoCONUT (k=2)

<bot> [Thought] [Thought] <eot>
Every rorpus is a bompus.
Alex is a bompus ✅

(Correct Path)

Continuous Space Enables Latent Tree Search



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Question: Is Alex a gorpus or bompus?

Ground Truth Solution

Alex is a grimpus.
Every grimpus is a rompus.
Every rompus is a bompus.
Alex is a bompus

Coconut (k=1)

<bot> [Thought] <eot>
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Every scrompus is a brimpus.
Alex is a brimpus ❌

(Wrong Target)

CoT

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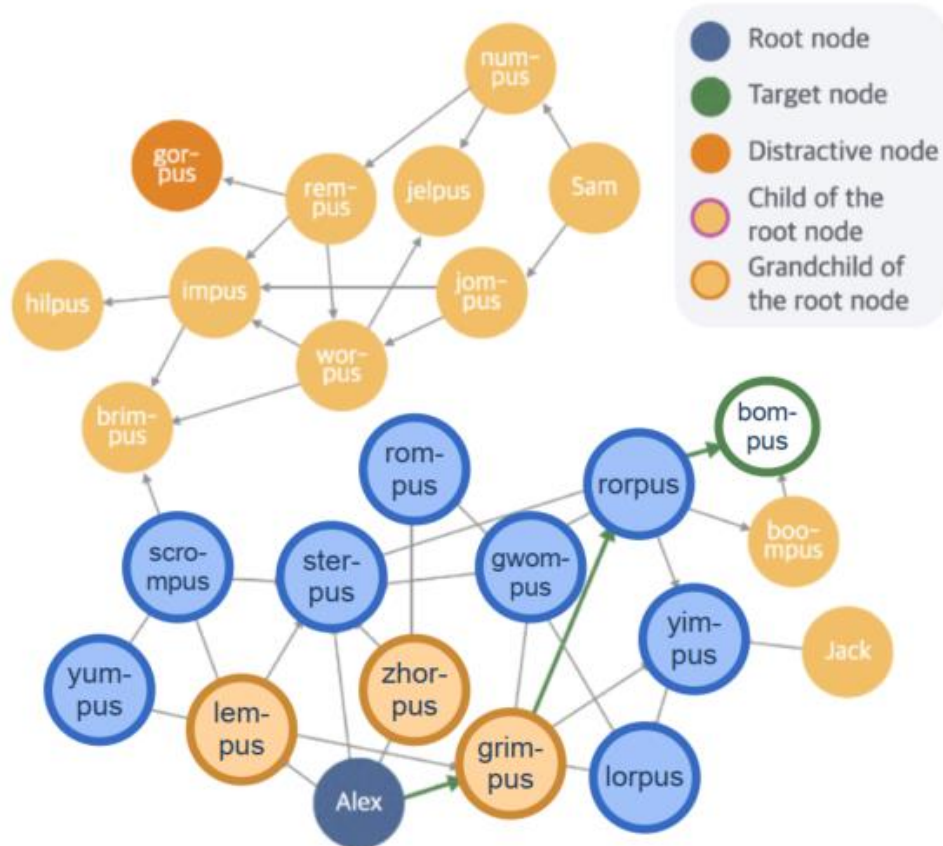
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Coconut (k=2)

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(Correct Path)

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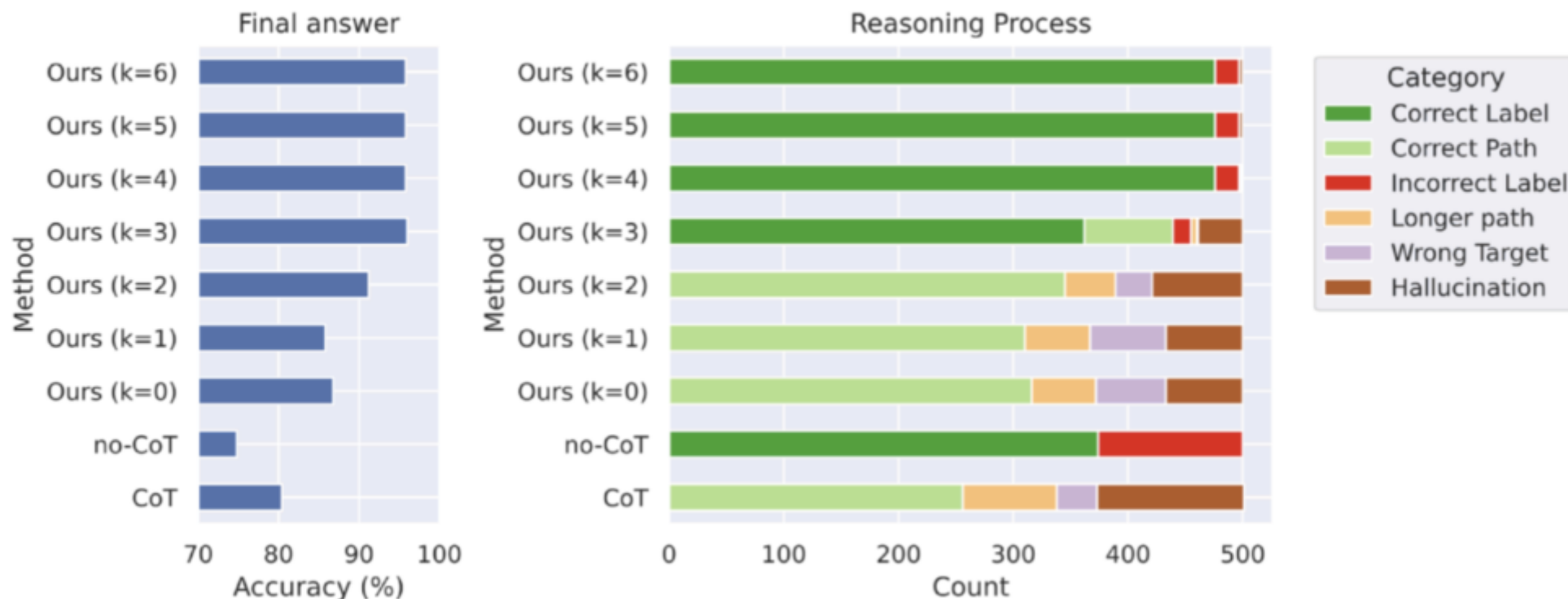
(Hallucination)

Coconut (k=2)

<bot> [Thought] [Thought] <eot>
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(Correct Path)

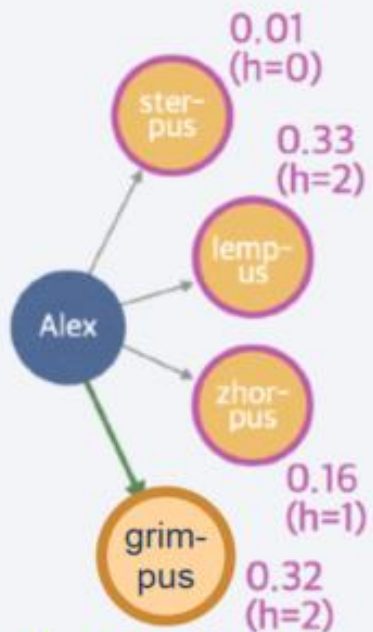
Interpolate between latent/language-space reasoning



K = #steps in **latent space**, the remaining steps in **language space**

Larger k -> **Higher accuracy** and **more correct reasoning process**

Latent thought as “superpositional state”

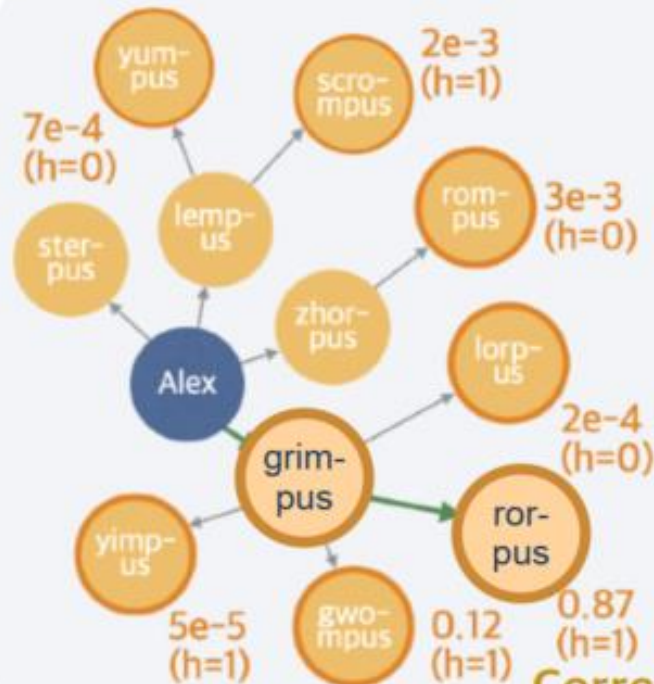


COCONUT (k=1)

<bot> [Thought] <eot>
Every lempus ...

$$\begin{aligned} p(\text{"lempus"}) \\ &= p(\text{"le"})p(\text{"mp"})p(\text{"us"}) \\ &= 0.33 \end{aligned}$$

Correct path



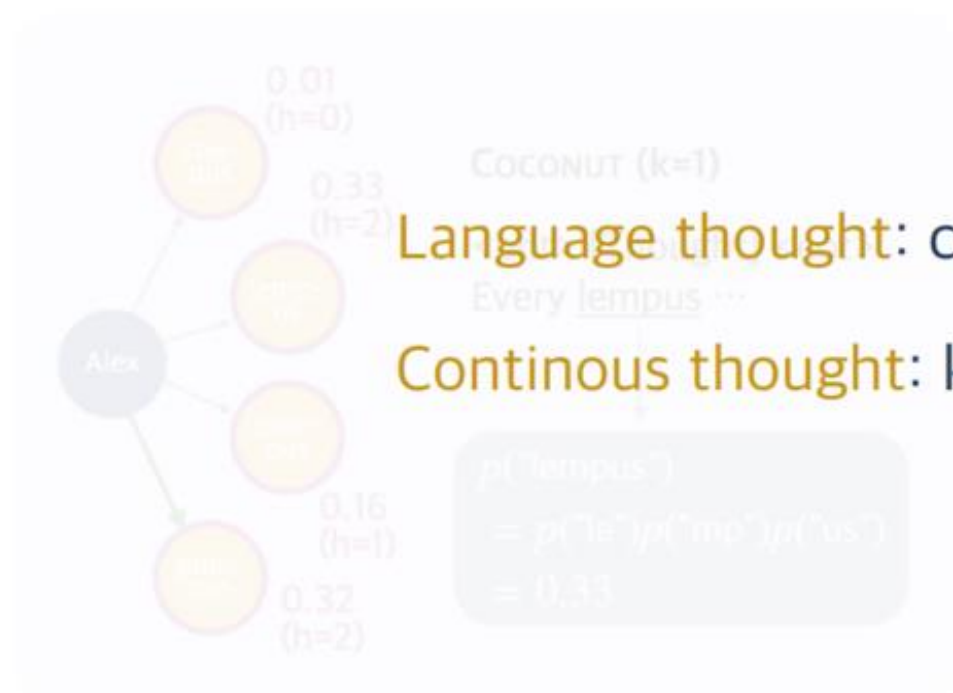
COCONUT (k=2)

<bot> [Thought] [Thought] <eot>
Every rorpus ...

$$\begin{aligned} p(\text{"rorpus"}) \\ &= p(\text{"ro"})p(\text{"rp"})p(\text{"us"}) \\ &= 0.87 \end{aligned}$$

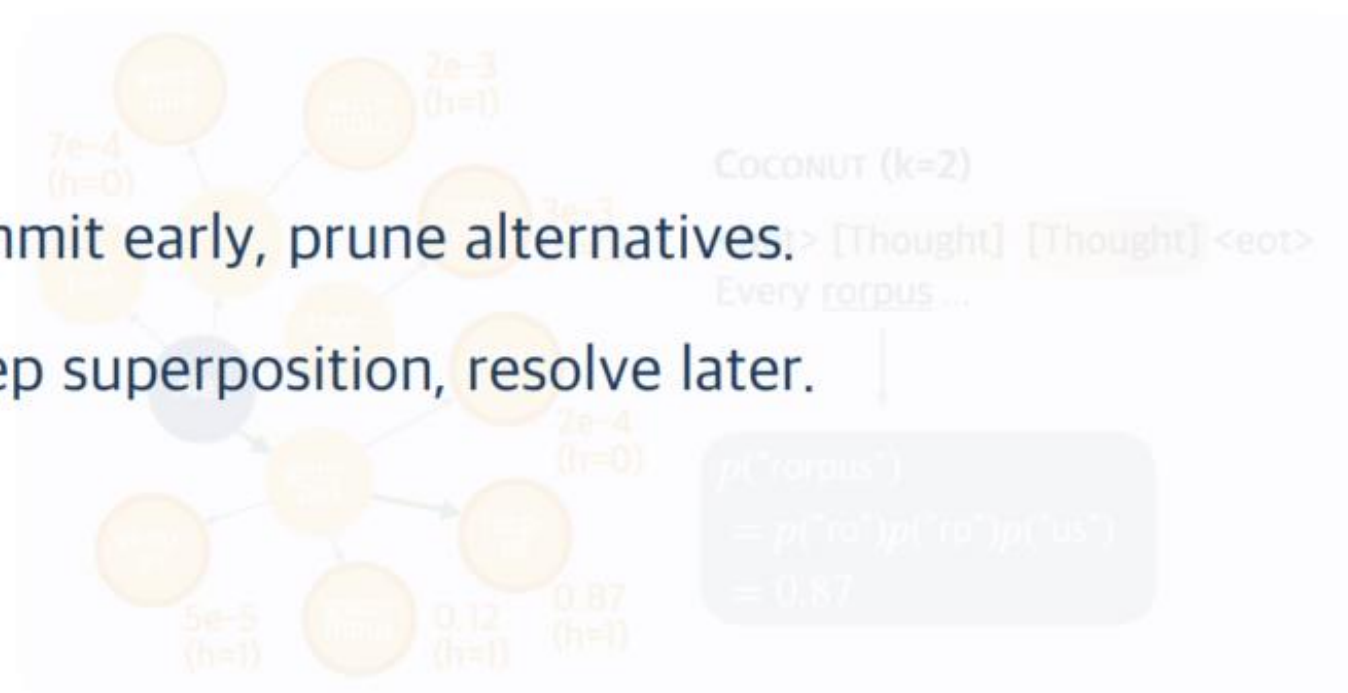
Correct path

Latent thought as “superpositional state”



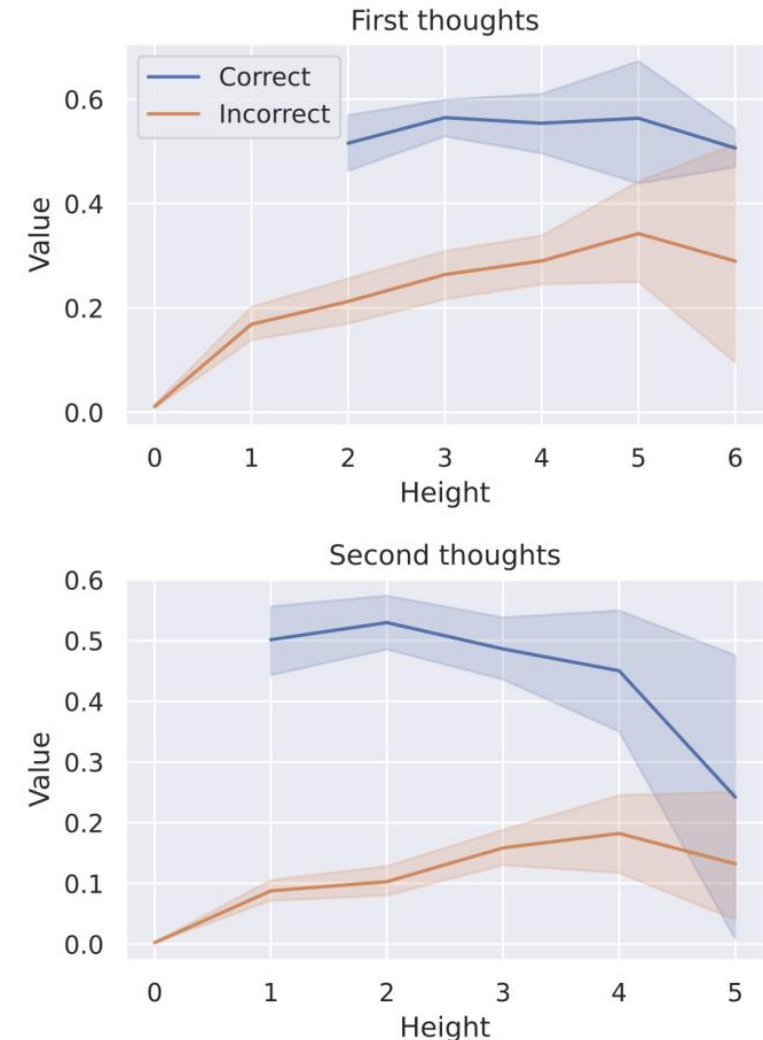
Language thought: commit early, prune alternatives.

Continuous thought: keep superposition, resolve later.

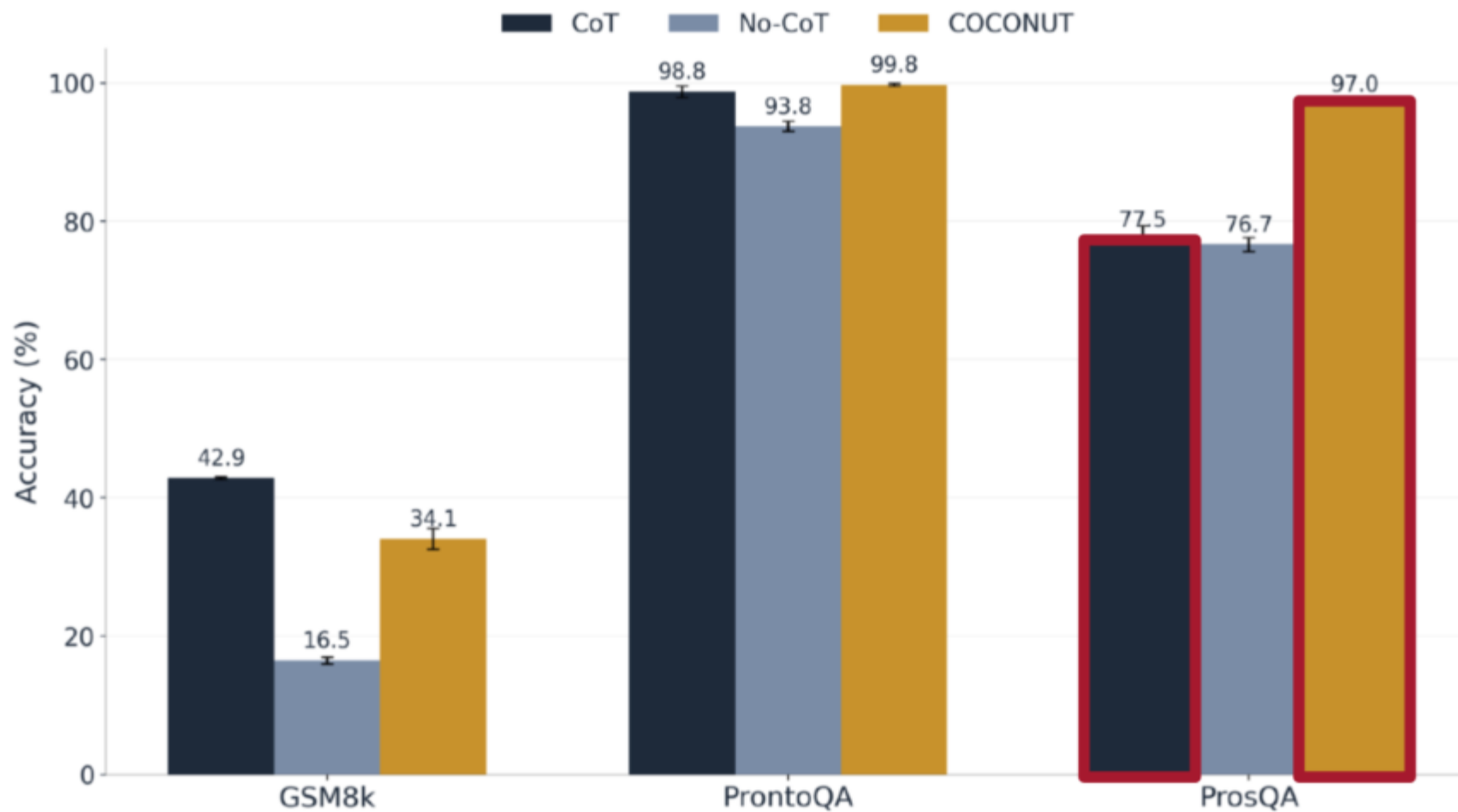


Why maintaining multiple candidate paths and postponing deterministic decisions enhances reasoning performance?

- Node height: shortest distance to any leaf node
- nodes with lower heights consistently receive more accurate and definitive probability evaluations
- nodes with greater heights exhibit more ambiguous evaluations, reflecting increased uncertainty.

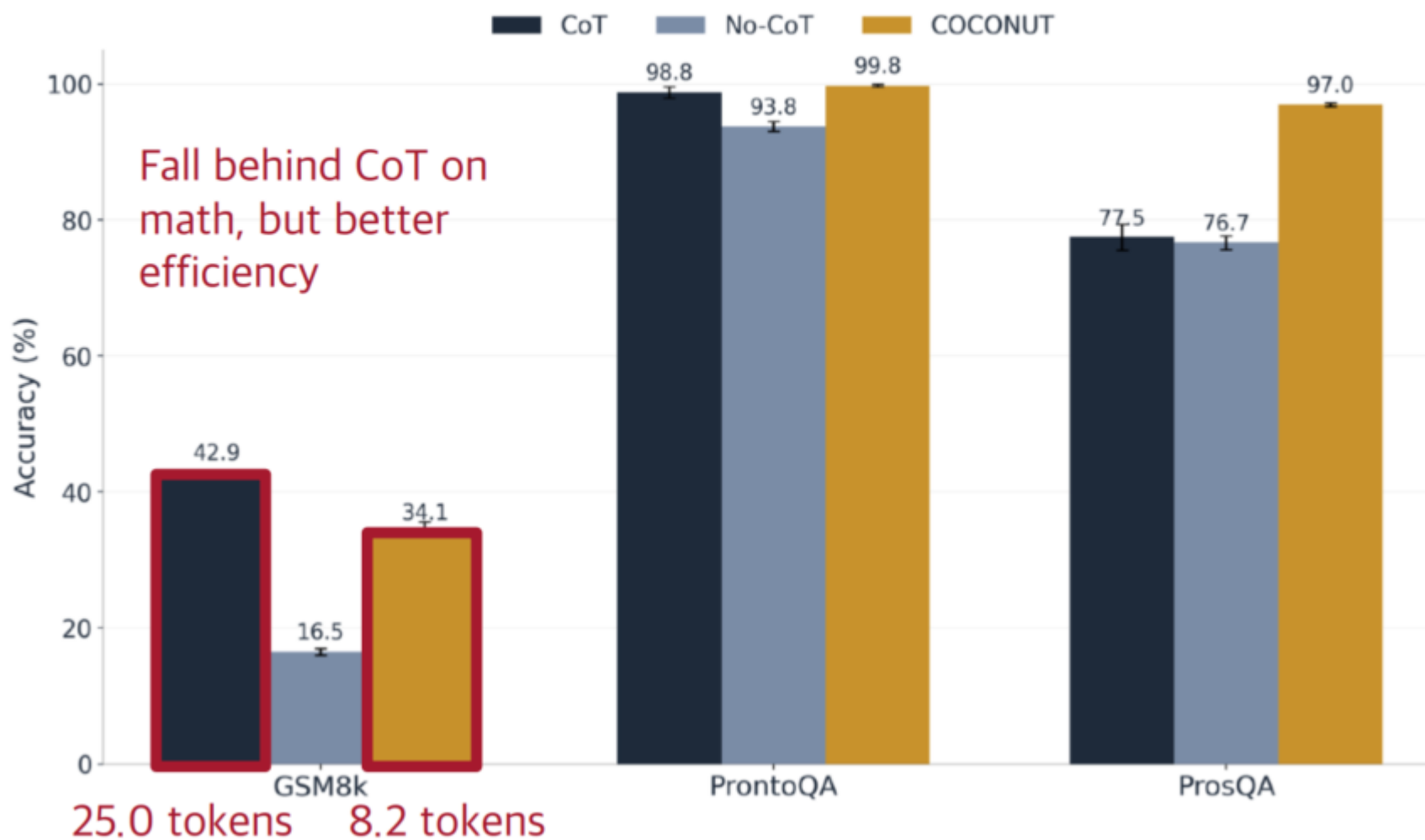


Evaluation



Better at problems with
larger branching factors

Evaluation



Conclusion

- “Chaining” continuous thoughts enhances reasoning.
- Continuous thoughts are efficient representations of reasoning.
- The LLM still needs guidance to learn latent reasoning.

Thanks for your listening!

Credit for this slide goes to the first author, Shibo Hao.

Questions?